MPZ 3231 - Assignment 4

(1) (i) Express in $a + i\,b$ form and find the modulus of

(a) $\frac{1+4i}{4+i}$ (b) $\frac{2-\sqrt{3}i}{1+i}$

(ii) If $\omega = 3\,i\,z - z^2$ and $z = x + i\,y$, Find $|\omega|^2$ in terms of x and y

(iii) Use De Moivre's Theorem to simplify the following expressions

(a) $\frac{\cos 7\theta + i \sin 7\theta}{\cos 2\theta - i \sin 2\theta}$

(b) $(\cos 2\theta + i \sin 2\theta)^2 . (\cos \theta + i \sin \theta)^3$

(2) Prove the following trignometric identities using methods based on De Moivre's theorem.

(a) $\tan 6\theta = 2t \left[\frac{3-10\,t^2+3\,t^4}{1-15\,t^2+15\,t^4-t^6}\right]$, where $t = \tan\theta$

(b) Prove that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ and hence solve the equation $1 - 3t^2 = 3\,t - t^3$ give your answers correct to three significant figures and verify these results by an algebraic method.

(c) Use De Moivre's theorem to find the following integrals.

$$\int (32\cos^6\theta - \cos 6\theta)\, d\theta$$

(d) Find the cube roots of -1. Show that they can be denoted by $-1, \lambda, -\lambda^2$ and prove that, $\lambda^2 - \lambda + 1 = 0$

(e) By considering the seventh root of -1, prove that $\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}$

(3) (i)The following table gives the details of the consumer preference for a new product to be introduced in the market:

	No. of consumers		
	Like	Dislike	Neutral
Male	500	250	125
Female	200	350	75

What is the probability that a consumer selected at random from the group will be;

(a) A male who disliked the product?

(b) One who liked the product, given that the person is a female?

(c) Either male or one who disliked the product?

(ii) The personnel department of a company has records which show the following analysis of its 200 engineers.

Age	Bachelor's degree	Master's degree	Total
Under 30	90	10	100
30 to 40	20	30	50
Over 40	40	10	50
Total	150	50	200

If one engineer is selected at random from the company, find;

(a) The probability he has only a bachelor's degree.

(b) The probability he has a master's degree, given that he is over 40.

(c) The probability he is under 30, given that he has only a bachelor's degree.

(4) Two players A and B are competing at a quiz game involving a series of questions. On any individual question, the probabilities that A and B give the correct answer are α and β respectively, for all questions, with outcomes for different questions being independent. The game finishes when a player wins by answering a question correctly.

Compute the probability that A wins if

(a) A answers the first question

(b) B answers the first question

(5) (i) Information is transmitted digitally as a binary sequence knows as "bits". However, noise on the channel corrupts the signal, in that a digit transmitted as a 0 is received as 1 with probability$1-\alpha$, with a similar random corruption when the digit 1 is transmitted. It has been observed that, across a large number of transmitted signals, the 0s and 1s are transmitted in the ratio 3:4

Given that the sequence 101 is received, what is the probability distribution over transmitted signals? Assume that the transmission and reception process are independent.

(ii) Suppose that a product is produced in three factories A, B and C. It is known that factory A produces twice as many items as factory B, and that factories B and C produce the same number of products. Assume that it is known that 2 percent of the items produced by each of the factories A and B are defective while 4 percent of those manufactured by factory C are defective. All the items produced in three factories are stocked, and an item of product is selected at random. What is the probability that this item is defective?